MATH-861 Advanced Topology

Credit Hours: 3-0
Prerequisite: Nil

Course Objectives: The course aims at developing an understanding about advanced concepts of Topology which are the basic tools of working mathematicians in a variety of fields. It covers some cover concepts including compactness and connectedness and explains how these concepts of Analysis are generalized to Topology.

Core Contents: Topological Spaces, Neighborhood, Bases, Initial & Final Topology, Quotient Spaces, Inadequacy of Sequences, Nets, Filters & Ultra Filters, Lindelöf Spaces, Separable Spaces, Compactness, Compactness in terms of filters, Locally Compact Spaces, One-point compactification, Stone-Cech Compactification, Para-compactness, Connectedness, Connected Components, Pathwise & Locally Connected Spaces,

Detailed Course Contents: Topological Spaces, Neighborhood, Neighborhood base, Subbases, Local Bases, Bases, Initial/Weak Topology and its Applications, Final/Strong Topology and its Applications, Quotient Spaces, Inadequacy of Sequences, Nets and their properties, Filters, Filter bases, Ultra Filters, Topology induced by filters, Relation b/w filters & Nets, Lindelöf Spaces, Separable Spaces, Compactness, Compactness in terms of Closedness & filters, Countable compactness, Limit-point compactness, One-point compactification, Stone-Cech compactifications, Connectedness, Connected components, Totally Disconnected spaces, Locally connected spaces and its applications, Pathwise connectedness and its relation to connectedness.

Course Outcomes: Upon completion of this course, the student should be able to:

- Continuous mappings, Disjoint Homeomorphism, Weak and Strong topologies, Quotientspaces
- Inadequacy of Sequences, Nets, Filters & Ultra Filters
- Lindelöf Spaces, Separable Spaces
- Compactness, Countable, Limit-point and local compactness
- One-point & Stone-Cech Compactifications
- Connectedness, Connected components, Totally disconnectedness, Pathwise & LocalConnectedness

Text Book: S. Willard, "General Topology", Dover Publications; Illustrated Edition, (2004)

Reference Books:

1. James R. Munkres, "Topology", Prentice, Hall, Inc., 2nd Edition (2000)
2. T. D. Bradley, T. Bryson, J. Terilla, "Topology: A Categorical Approach", MIT

Press, (2020)
3. G. Preuss, “Foundations of Topology: An Approach to Convenient Topology”, Springer, 2ndEdition, (2002).
4. J. Kelly, “General Topology”, Springer, (2005).

ASSESSMENT SYSTEM

Nature of assessment	Frequency	Weightage (%age)
Quizzes	Minimum 3	10-15
Assignments	-	5-10
Midterm	1	25-35
End Semester Examination	1	40-50
Project(s)	-	10-20

Weekly Breakdown

Week	***Section***	***Topics***
1	**Sec. 3-4**	Review of Topological spaces and Examples, Neighborhood operators,Topology induced by neighborhoods, Neighborhood bases, Open, closed, interiors and closures in terms of neighborhoods
2	**Sec. 5-6**	Subbases, Bases, Local bases and their properties, Subspaces and its properties, and related results
3	**Sec. 7**	Continuous functions between topologies, and their characterizations using neighborhood operators, characterizations of spaces using continuous mappings, Continuous functions to and from a plane., Disjoint homeomorphisms
4	**Sec.8**	Weak Topologies and their applications, Box products and their related results, Tychonoff Topologies
5	**Sec. 9**	Strong/Final Topologies and their applications, Quotient spaces, identification spaces, Quotients vs Decompositions
6	**Sec. 10**	Inadequacy of sequences, sequentially convergences, 1st, and 2nd countable and its applications
7	**Sec. 11**	Nets, Ultra nets and their examples, subnets and related results, Net convergence in topologies
8	**Sec. 12**	Filters, Ultrafilters, Topologies induced by filters, Filter convergence in topological spaces, Relationship between filters and nets
9	**Mid Semester Exam**	

10	Sec.13-14	Lower Separation axioms and related results, Regular and completely regular spaces
11	Sec. 15-16	Normal spaces and related results, Urysohn Lemma and Tietze Extension Theorem, Shrinkable spaces, Separable and Lindelöf spaces and Results
12	Sec. 17	Compactness, Compactness in terms of neighborhoods and filters, sequentially compactness and their related results, Countable compactness,and related theorems
13	Sec. 18	Locally compact spaces, examples and its relations with compactness,countable compactness and sequentially compactness, and their related results
14	Sec. 19	Compactifications, Alexandroff Compactifications, Stone-Cech Compactifications
15	Sec. 26	Connectedness and examples, Connectedness in terms of neighborhood and filters, Mutual Separated spaces, Connected components and their relatedresults
16	Sec. 27	Pathwise connectedness and locally connectedness, examples and their related results and their relation with connectedness and mutual separateness
17	Sec. 29	Totally disconnected spaces, examples and related results, Zero-dimensional spaces, examples, and related theorems.
18	**End Semester Exam**	